

# MA0001 Øving 5

Øystein Tveit



## Innhold

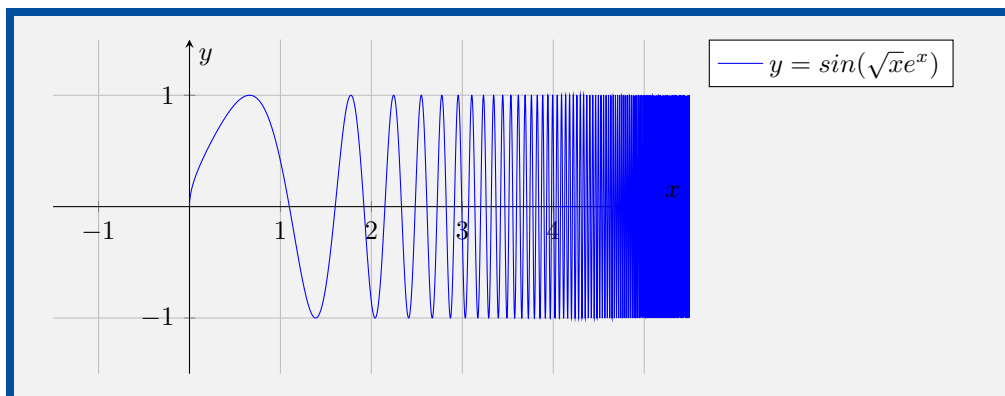
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## 1 Forberedende oppgaver

1 a)

$$f(x) = \sin(\sqrt{x}e^x)$$

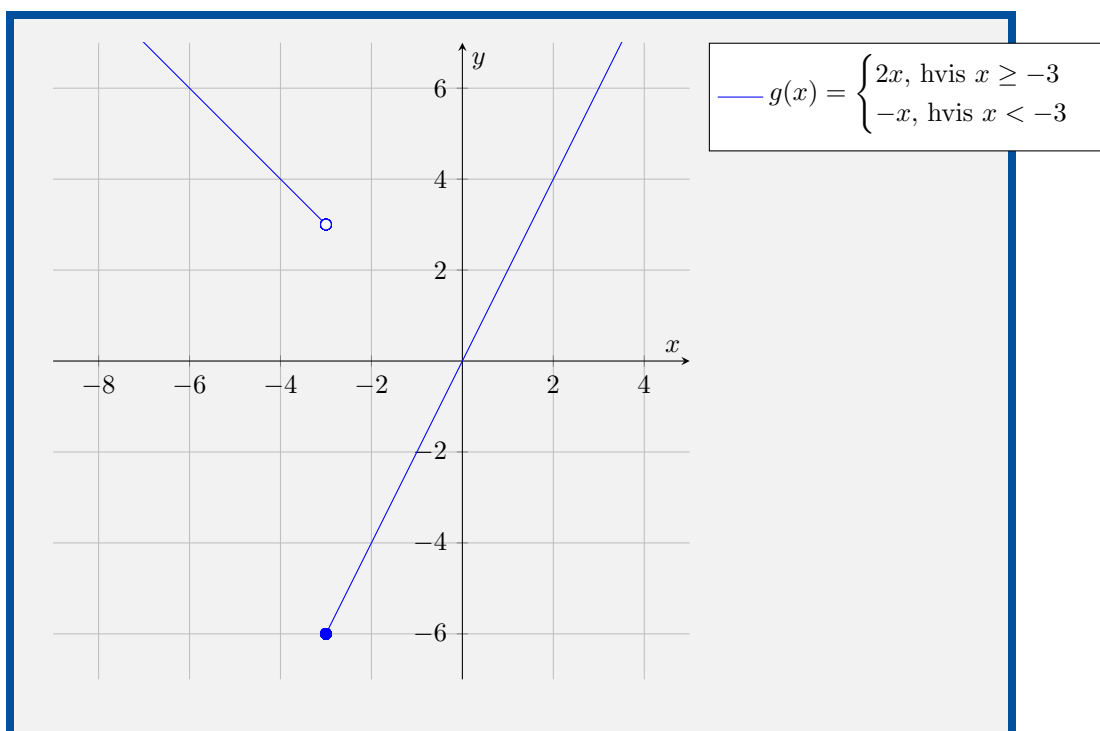
er kontinuerlig for alle punkt hvor den er definert



b)

$$g(x) = \begin{cases} 2x, & \text{hvis } x \geq -3 \\ -x, & \text{hvis } x < -3 \end{cases}$$

er ikke kontinuerlig ved  $x = -3$ , hvor  $y$ -verdien gjør et hopp fra 3 til  $-6$



## 2 Innleveringsoppgaver

2

$$\begin{aligned}\lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 6n + 9}}{\sqrt{n^2 + 1}} \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{n^2 + 6n + 9}{n^2 + 1}}\end{aligned}$$

Ettersom  $\lim_{n \rightarrow \infty} (a_n b_n) = (\lim_{n \rightarrow \infty} a_n)(\lim_{n \rightarrow \infty} b_n)$

$$\lim_{n \rightarrow \infty} a_n = \sqrt{\lim_{n \rightarrow \infty} \frac{n^2 + 6n + 9}{n^2 + 1}}$$

Ettersom  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n}$

I tillegg deler vi både teller og nevner på  $n^2$

$$\lim_{n \rightarrow \infty} a_n = \sqrt{\frac{\lim_{n \rightarrow \infty} 1 + \frac{6}{n} + \frac{9}{n^2}}{\lim_{n \rightarrow \infty} 1 + \frac{1}{n^2}}}$$

Både  $\frac{9}{n^2}$ ,  $\frac{6}{n}$  og  $\frac{1}{n^2}$  går mot 0 når  $n \rightarrow \infty$

$$\begin{aligned}\lim_{n \rightarrow \infty} a_n &= \sqrt{\frac{1 + 0 + 0}{0 + 1}} \\ &= \sqrt{1} \\ &= 1\end{aligned}$$

3

$$\begin{aligned}\lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} (\sqrt{n^2 - n + 9} - \sqrt{n^2 + 9}) \\ &= \lim_{n \rightarrow \infty} (\sqrt{n^2 - n + 9} - \sqrt{n^2 + 9}) \frac{\sqrt{n^2 - n + 9} + \sqrt{n^2 + 9}}{\sqrt{n^2 - n + 9} + \sqrt{n^2 + 9}} \\ &= \lim_{n \rightarrow \infty} \frac{(n^2 - n + 9) - (n^2 + 9)}{\sqrt{n^2 - n + 9} + \sqrt{n^2 + 9}} \\ &= \lim_{n \rightarrow \infty} \frac{-n}{\sqrt{n^2 - n + 9} + \sqrt{n^2 + 9}} \\ &= - \lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 - n + 9} + \sqrt{n^2 + 9}} \\ &= - \lim_{n \rightarrow \infty} \frac{1}{n^{-1}\sqrt{n^2 - n + 9} + n^{-1}\sqrt{n^2 + 9}} \\ &= - \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^{-2}(n^2 - n + 9)} + \sqrt{n^{-2}(n^2 + 9)}} \\ &= - \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 - \frac{1}{n} + \frac{9}{n^2}} + \sqrt{1 + \frac{9}{n^2}}}\end{aligned}$$

Dermed blir

$$\begin{aligned} -\lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 - \frac{1}{n} + \frac{9}{n^2}} + \sqrt{1 + \frac{9}{n^2}}} &= -\frac{1}{\sqrt{1 - 0 + 0} + \sqrt{1 + 0}} \\ &= -\frac{1}{1 + 1} \\ &= -\frac{1}{2} \end{aligned}$$

4

$$a_{k+1} = 2 - \frac{1}{a_k}$$

Fikspunktet vil være punktet hvor  $a_{k+1} = a_k$

$$\begin{aligned} a &= 2 - \frac{1}{a} \\ a - 2 &= -\frac{1}{a} \\ a^2 - 2a &= -1 \\ a^2 - 2a + 1 &= 0 \\ (a - 1)^2 &= 0 \quad \Leftrightarrow \quad a = 1 \end{aligned}$$

$a$  har ett fikspunkt ved  $a = 1$

5

$$\begin{cases} b_0 = 1 \\ b_1 = 2 \\ b_{n+1} = b_n + 2 \cdot b_{n-1} \end{cases}$$

$$\begin{aligned} b_2 &= b_1 + 2 \cdot b_0 = 2 + 2 \cdot 1 = 4 \\ b_3 &= b_2 + 2 \cdot b_1 = 4 + 2 \cdot 2 = 8 \\ b_4 &= b_3 + 2 \cdot b_2 = 8 + 2 \cdot 4 = 16 \\ b_5 &= b_4 + 2 \cdot b_3 = 16 + 2 \cdot 8 = 32 \\ b_6 &= b_5 + 2 \cdot b_4 = 32 + 2 \cdot 16 = 64 \\ b_7 &= b_6 + 2 \cdot b_5 = 64 + 2 \cdot 32 = 128 \end{aligned}$$

Jeg gjetter at  $f(n) = 2^n$ .

$$\begin{aligned} b_n + 2 \cdot b_{n-1} &= 2^n + 2 \cdot 2^{n-1} \\ &= 2^n + 2^n \\ &= 2 \cdot 2^n \\ &= 2^{n+1} \\ &= b_{n+1} \end{aligned}$$