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Innhold

1 Forberedende oppgaver **2**
Oppgave 1 2

2 Innleveringsoppgaver **3**
Oppgave 2 3
Oppgave 3 4
Oppgave 4 5
Oppgave 5 5

1 Forberedende oppgaver

1

$$f(x) = \frac{1}{2}x^3 - 7x + 22$$

$$f'(x) = \frac{3}{2}x^2 - 7$$

$$f''(x) = 3x$$

$$f'''(x) = 3$$

$$f^{(100)} = 0$$

2 Innleveringsoppgaver

2 1.

$$f(x) = ln\left(\frac{1}{x^2}\right)$$

La $u = \frac{1}{x^2}$
Jeg bruker kjerneregelen:

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx}\\ &= \frac{d}{du}ln(u) \cdot \frac{d}{dx}\frac{1}{x^2}\\ &= \frac{1}{u} - 2\frac{1}{x^3}\\ &= \frac{1}{\frac{1}{x^2}} - 2\frac{1}{x^3}\\ &= x^2 - 2\frac{1}{x^3}\end{aligned}$$

2.

$$g(x) = \frac{1 + \sin x}{1 + e^x + x^2}$$

Jeg bruker kvotientregelen:
$u = 1 + \sin x$
$v = 1 + e^x + x^2$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{d}{dx}(u) \cdot v - u \cdot \frac{d}{dx}(v)}{v^2}$$

$$\begin{aligned}\frac{d}{dx}u &= \frac{d}{dx}1 + \sin x\\ &= \cos x\end{aligned}$$

$$\begin{aligned}\frac{d}{dx}v &= \frac{d}{dx}1 + e^x + x^2\\ &= e^x + 2x\end{aligned}$$

$$\frac{dg}{dx} = \frac{(\cos x)(1 + e^x + x^2) - (1 + \sin x)(e^x + 2x)}{(1 + e^x + x^2)^2}$$

3.

$$h(x) = \sqrt{1 + \sqrt{x}}$$

$u = 1 + \sqrt{x}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dv} \cdot \frac{du}{dx} \\ &= \frac{d}{du}\sqrt{u} \cdot \frac{d}{dx}\left(1+\sqrt{x}\right) \\ &= \frac{1}{2\sqrt{u}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{1+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{4\sqrt{1+\sqrt{x}}\sqrt{x}}\end{aligned}$$

3

$$f(t) = \frac{t^2-1}{t+1} + 6t^{1/3} + \sqrt{\sin t} + 4^t$$

Jeg deriverer funksjonen ledd for ledd

Ledd 1:

$$\begin{aligned}\frac{d}{dt}\frac{t^2-1}{t+1} &= \frac{d}{dt}\frac{(t+1)(t-1)}{t+1} \\ &= \frac{d}{dt}t-1 \\ &= 1\end{aligned}$$

Ledd 2:

$$\begin{aligned}\frac{d}{dt}6t^{1/3} &= \frac{1}{3} \cdot 6t^{(1/3-1)} \\ &= 2t^{-2/3} \\ &= \frac{2}{\sqrt[3]{t^2}}\end{aligned}$$

Ledd 3:

$$\frac{d}{dt}\sqrt{\sin t}$$

$u = \sin t$

$$\begin{aligned}\frac{dy}{dt} &= \frac{dy}{du} \cdot \frac{du}{dt} \\ &= \frac{d}{du}\sqrt{u} \cdot \frac{d}{dt}\sin t \\ &= \frac{1}{2\sqrt{u}} \cdot \cos t \\ &= \frac{\cos t}{2\sqrt{\sin t}}\end{aligned}$$

Ledd 4:

$$\frac{d}{dt}4^t = 4^t ln(t)$$

$$\frac{df}{dx} = 1 + \frac{2}{\sqrt[3]{t^2}} + \frac{\cos t}{2\sqrt{\sin t}} + 4^t ln(t)$$

4 Her har jeg brukt python for å løse oppgaven.

```python
from math import e

def f(x): return e ** x - 2
def df(x): return e ** x

def Newton(a, f, df):
  return a - f(a)/df(a)

def main():
  i = 0
  a = [5]
  while True:
    a.append(Newton(a[i], f, df))
    if abs(a[i] - a[i-1]) < 0.001:
      print(f'Iterasjoner: {i-1}, a: {a[i-1]}')
      break
    i += 1

main()
```

./scripts/4.py

Output: Iterasjoner: 7, a: 0.6932882713164431

5

$$f(y) = a^y$$

$$f'(y) = a^y \ln a$$

$$f^{-1}(y) = \log_a y$$

$$\begin{aligned}\frac{d}{dy}f^{-1}(y) &= \frac{1}{f' \circ f^{-1}(y)} \\ &= \frac{1}{a^{\log_a y} \ln a} \\ &= \frac{1}{y \ln a}\end{aligned}$$

Gitt at vi vet at $\frac{d}{dx}ln(x) = \frac{1}{x}$, så kan vi også løse det på følgende måte

$$\begin{aligned}\frac{d}{dy}\log_a y &= \frac{d}{dy}\frac{\ln y}{\ln a}\\ &= \frac{1}{\ln a}\cdot\frac{d}{dy}\ln y\\ &= \frac{1}{\ln a}\cdot\frac{1}{y}\\ &= \frac{1}{y\ln a}\end{aligned}$$