these are my solutions to the seventh exercise set of TMA4135.

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exercise 7

problem 1

a)

$$\begin{cases} 5y + 2y' + y'' + 3 = 0 \\ y(0) & = 1 \\ y'(0) & = 4 \end{cases}$$

we introduce the substitutions $y_1 = y$ and $y_2 = y_1' = y$ such that we get

$$5y_1 + 2y_2 + {y_2}' + 3 = 0$$
$$\Rightarrow {y_2}' = y'' = -3 - 2y_2 - 5y_1$$

for the first equation. this yields an autonomous first-order system

$$\begin{cases} {y_1}' = y_2 \\ {y_2}' = -3 - 2y_2 - 5y_1 \end{cases}$$

since there is no explicit t on the right sides of the equations, i.e. the state of the system only relies on its current state.

since we only use a step size of $h = 1$, we simply do

$$y_{n+1} = y_n + y_n',$$

so starting at $t = 0$ we get, using our initial values

$$\begin{cases} y_1(0) = 1 \quad \text{initial} \\ y_2(0) = 4 \quad \text{initial} \\ y_1'(0) = y_2(0) = 4 \\ y_2'(0) = -3 - 2y_2(0) - 5y_1(0) = -16 \end{cases}$$

which we can plug in to get our next values for y_1 and y_2

$$\begin{cases} y_1(1) = y_1(0) + y_1'(0) = 5 \\ y_2(1) = -12 \end{cases}$$

which concludes the first step in euler's method.

b)

for $k = 0, 1, 2, 3$

$$\begin{cases} y + 3y'' + 4y^{(3)} + \sin(t) = 3 \\ y^{(k)}(0) & = 4 - k \end{cases}$$

yields five equations.

in this case, we can make yet more substitutions

$$\begin{cases} y_0 = y \\ y_k' = y_{k+1} = y^{(k+1)} \end{cases}$$

such that

$$\begin{cases} y_0 + 3y_2 + 4y_3 + \sin(t) = 3 \\ y^{(k)}(0) = 4 - k \end{cases}$$

$$\Rightarrow \begin{cases} y_0' = y_1 \\ y_1' = y_2 \\ y_2' = y_3 \\ y_3' = -\frac{1}{4}(y_1 + 3y_3 + \cos(t)) \end{cases}$$

so starting at $t = 0$ with step size $h = 1$ we obtain

$$y_0(0) = 4, \quad y_1(0) = 3, \quad y_2(0) = 2, \quad y_3(0) = 1$$

and

$$y_0'(0) = 3, \quad y_1'(0) = 2, \quad y_2'(0) = 1, \quad y_3'(0) = -7/4$$

such that after one step we have

$$\begin{cases} y_0(1) = y_0(0) + y_0'(1) = 7 \\ y_1(1) = 5 \\ y_2(1) = 3 \\ y_3(1) = -3/4 \end{cases}$$