TMA4135 - Exercise 6

Problem 1

Define convolution as

$$f * g := \int_0^t f(\tau) g(t-\tau)\, d\tau$$

$$= \int_0^t g(\tau) f(t-\tau)\, d\tau .$$

a) i) $e^t * e^{-t} = \int_0^t e^{\tau} \cdot e^{\tau - t}\, d\tau$

$$= \int_0^t e^{2\tau - t}\, d\tau , \quad u = 2\tau - t \Rightarrow du = 2 d\tau$$

$$= \frac{1}{2}\int_{-t}^t e^u\, du = \frac{1}{2}\left[e^u\right]_{-t}^{t} = \underbrace{\frac{1}{2}\left[e^t - e^{-t}\right]}_{= \sinh t}$$

ii) $\sin(\omega t) * \cos(\omega t)$

$$= \int_0^t \underline{\sin(\omega\tau) \cdot \cos(\omega[t-\tau])}\, d\tau$$

$$= \sin(\omega[\cancel{\tau} + t - \cancel{\tau}]) + \sin(\omega[\tau - t + \tau])$$

$$= \sin(\omega t) + \sin(\omega[2\tau - t])$$

$$= \int_0^t \overbrace{\sin(\omega t)}^{\text{const.}}\, d\tau + \int_0^t \sin(\omega[2\tau - t])\, d\tau$$

$$= t \sin(\omega t) + \frac{1}{2\omega}\int_{-\omega t}^{\omega t} \sin u\, du$$

$$= \frac{1}{2\omega}\left[-\cos u\right]_{-\omega t}^{\omega t} = \frac{1}{2\omega}\left(\cos(-\omega t) - \cos(\omega t)\right)$$

$$= t\sin(\omega t) + \frac{1}{2\omega}\Big(\underbrace{\cos(\cancel{-}\omega t)}_{\substack{\cos(-\theta)\\ =\cos(\theta)}} - \cos(\omega t)\Big).$$

$$= \underline{\underline{t\sin(\omega t)}}$$

b) i) $y(t) - \int_0^t y(\tau)\,d\tau = 1$

$$\Leftrightarrow y(t) - (y * 1)(t) = 1$$

$$\Leftrightarrow (y * 1)(t) = y(t) - 1$$

$$\Leftrightarrow \mathcal{L}(y * 1) = \mathcal{L}(y-1)$$

$$\Leftrightarrow \mathcal{L}(y)\cdot\mathcal{L}(1) = \mathcal{L}(y) - \mathcal{L}(1)$$

$$\Leftrightarrow Y/s = Y - 1/s$$

$$\Rightarrow Y = \frac{1}{s-1} \Rightarrow \underline{\underline{y = e^t}}$$

ii) $y(t) - \int_0^t y(\tau)\cosh(t-\tau)\,d\tau = t + e^t$

$$\Leftrightarrow y(t) - (y * \cosh)(t) = t + e^t$$

$$\Leftrightarrow (y * \cosh)(t) = y(t) - t - e^t$$

$$\Leftrightarrow \mathcal{L}(y)\cdot\mathcal{L}(\cosh) = \mathcal{L}(y) - \mathcal{L}(t) - \mathcal{L}(e^t)$$

$$\Leftrightarrow Y\cdot\frac{s}{s^2-1} = Y - \frac{1}{s^2} - \frac{1}{s-1}$$

$$\Rightarrow Y(s) = -\frac{1/s^2 + 1/(s-1)}{s/(s^2-1) - 1}$$

$$= \frac{s-(s^2-1)}{s^2-1} = \frac{1+s-s^2}{s^2-1}$$

$$= \frac{s-1+s^2}{s^2(s-1)} = \frac{s^2+s-1}{s^3-s^2}$$

$$= - \frac{s^2+s-1}{s^3-s^2} \cdot \frac{s^2-1}{1+s-s^2}$$

$$= - \frac{s^4+s^3-s^2-s^2-s+1}{\cancel{s^3}+s^4-s^5-s^2-\cancel{s^3}+s^4}$$

$$= \frac{-s^4-s^3+2s^2+s-1}{-s^5+2s^4-s^2}$$

$$= (-s^3+2s^2-1)s^2$$

Try $s=1 \rightarrow s=1$ is a root, so

$$(-s^3+2s^2-1):(s-1) = -s^2+s+1$$

$$\begin{array}{l} -(-s^3+s^2) \\ \hline s^2 \\ -(s^2-s) \\ \hline s-1 \\ -(s-1) \\ \hline 0 \end{array}$$

$\downarrow$ only complex roots

$$= (-s^2+s+1)(s-1)s^2$$

$$= \frac{-s^4-s^3+2s^2+s-1}{(-s^2+s+1)(s-1)s^2}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{-s^2+s+1}$$

$$\Longleftrightarrow -s^4-s^3+2s^2+s-1$$

$$
\left.\begin{aligned}
= \; & A(-s^2+s+1)(s-1)s \\
+ \; & B(-s^2+s+1)(s-1) \\
+ \; & C(-s^2+s+1)s^2 \\
+ \; & D(s-1)s^3 \\
+ \; & E(s-1)s^2
\end{aligned}\right\}
\begin{array}{c}
\\ s^4 \\ s^3 \\ s^2 \\ s \\ 1
\end{array}
\begin{array}{c}
\begin{array}{cccccc} A & B & C & D & E & = P \end{array} \\
\left[\begin{array}{ccccc|c}
-1 & 0 & -1 & 1 & 0 & -1 \\
2 & -1 & 1 & -1 & 1 & -1 \\
0 & 2 & 1 & 0 & -1 & 2 \\
-1 & 0 & 0 & 0 & 0 & 1 \\
0 & -1 & 0 & 0 & 0 & -1
\end{array}\right]
\end{array}
$$

$$
\overset{\text{WA}}{\Longrightarrow} [A \; B \; C \; D \; E]^T = [-1 \; 1 \; 0 \; -2 \; 0]
$$

$$
\Longrightarrow Y(s) = -\frac{1}{s} + \frac{1}{s^2} - \frac{2s}{-s^2+s+1}
$$

$$
= 2 \cdot \frac{s}{s^2-s-1} = 2 \cdot \frac{s - 1/2 + 1/2}{(s-1/2)^2 - 5/4}
$$

$$
= 2 \left[\frac{s-1/2}{(s-1/2)^2 - (\sqrt{5}/2)^2} + \frac{\sqrt{5}}{5} \cdot \frac{\sqrt{5}/2}{(s-1/2)^2 - (\sqrt{5}/2)^2} \right]
$$

$$
\Longrightarrow y(t) = -1 + t
$$

$$
+ 2e^{1/2 t}\left[\cosh\left(\frac{\sqrt{5}}{2}t\right) + \frac{\sqrt{5}}{5}\sinh\left(\frac{\sqrt{5}}{2}t\right)\right]
$$

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iii) $y(t) - \int_0^t y(\tau)(t-\tau)\,d\tau = 2 - \frac{t^3}{2}$

$$
\Longleftrightarrow y(t) - (y * t)(t) = 2 - \frac{t^2}{2}
$$

$$
\Longleftrightarrow (y * t)(t) = y(t) - 2 + \frac{t^2}{2}
$$

$$
\Longleftrightarrow Y \cdot \frac{1}{s^2} = Y - \frac{2}{s} + \frac{1}{s^3}
$$

$$
\Longleftrightarrow Y s = Y s^3 - 2s^2 + 1
$$

$$\Leftrightarrow Y = \frac{1-2s^2}{s-s^3} = \frac{1-2s^2}{s(1+s)(1-s)} = \frac{1-2s^2}{s(s-1)(s+1)}$$

$$= \frac{-1}{s} - \frac{1}{2(s-1)} - \frac{1}{2(s+1)}$$

$$\Leftrightarrow y(t) = -1 - \frac{1}{2}e^{t} - \frac{1}{2}e^{-t}$$

$$\underline{\underline{= -1 - \cosh t}}$$

c) i) $$F(s) = \frac{2\pi s}{(s^2+\pi)^2} = \frac{s}{s^2+\pi} \cdot \frac{2\pi}{s^2+\pi}$$

$$= \frac{s}{s^2+\pi} \cdot \left[\frac{2\pi}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{s^2+\pi}\right]$$

$$= 2\sqrt{\pi}\left[\frac{s}{s^2+\pi} \cdot \frac{\sqrt{\pi}}{s^2+\pi}\right]$$

$$\Leftrightarrow f(t) = 2\sqrt{\pi}\left[\cos(\sqrt{\pi}t) * \sin(\sqrt{\pi}t)\right]$$

$$\overset{\text{magic}}{=} \underline{\underline{\frac{1}{2} t \sin(\sqrt{\pi} t)}}$$

ii) $$F(s) = \frac{e^{-as}}{s(s-2)} = e^{-as} \cdot \frac{1}{s} \cdot \frac{1}{s-2}$$

$$\Rightarrow f(t) = \delta(t-a) * 1 * e^{2t}$$

<u>Recall</u> that convolving with $\delta(t-a)$ is the same as shifting the function by a

$$f(t) * \delta(t-a) = f(t-a).$$

Additionally, we have $f(t) * 1 = \int_0^t f(\tau)\,d\tau$, and convolutions are commutative and associative, so

$$f(t) = \left(e^{2(t-a)} * 1\right)(t) = \int_0^t e^{2(\tau-a)}\, d\tau$$
$$= \int_a^{t+a} e^{2\tau}\, d\tau$$
$$= \frac{1}{2}\left[e^{u}\right]_{2a}^{2t+2a}$$
$$= \frac{1}{2}\left(e^{2t+2a} - e^{2a}\right)$$

(iii) $$F(s) = \frac{240}{(s^2+1)(s^2+25)} = \frac{1}{s^2+1} \cdot \frac{5}{s^2+25} \cdot 48$$

$$\Rightarrow f(t) = 48\left\{\sin t * \sin(5t)\right\}(t)$$
$$= 48 \int_0^t \sin\tau \cdot \sin\left[5(t-\tau)\right] d\tau$$
$$= 24 \int_0^t \left[\cos(\tau - 5t + 5\tau) - \cos(\tau + 5t - 5\tau)\right] d\tau$$
$$= 24 \left[\int_0^t \cos(6\tau - 5t)\, d\tau - \int_0^t \cos(5t - 4\tau)\, d\tau\right]$$
$$= 24\left(\frac{1}{6}\left[\sin u\right]_{-5t}^{t} + \frac{1}{4}\left[\sin u\right]_{5t}^{t}\right)$$
$$= 4\left(\sin t - \sin(-5t)\right) - 6\left(\sin(5t) - \sin t\right)$$
$$= 10 \sin t - 2\sin(5t)$$

Problem 2

a) $y(t) + 3t * y(t) = 2\cos t$

$$\overset{\mathcal{L}}{\Longrightarrow} Y + \frac{3}{s^2} \cdot Y = 2 \cdot \frac{s}{s^2+1}$$

$$\Longleftrightarrow Y = 2 \cdot \frac{s}{s^2+1} \cdot \frac{1}{1+3/s^2}$$

$$= 2 \cdot \frac{s}{s^2+1} \cdot \frac{s^2}{s^2+3}$$

$$= 2 \cdot \frac{s^3}{s^4+4s^2+3}$$

$$s^4+4s^2+3 = 0 \sim t^2+4t+3$$

$$= (t+3)(t+1) = 0$$

$$\Longrightarrow = 2 \cdot \left[\frac{As+B}{s^2+3} + \frac{Cs+D}{s^2+1} \right]$$

$$\Longleftrightarrow s^3 = (As+B)(s^2+1) + (Cs+D)(s^2+3)$$

$$= A(s^3+s) + B(s^2+1)$$

$$+ C(s^3+3s) + D(s^2+3)$$

$$\Longrightarrow \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} 1 \\ s \\ s^2 \\ s^3 \end{array} & \left[\begin{array}{cccc|c} 0 & 1 & 0 & 3 & 0 \\ 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{array}\right] \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\Longrightarrow Y = \frac{3s}{s^2+3} - \frac{s}{s^2+1} = 3 \cdot \frac{s}{s^2+3} - \frac{s}{s^2+1}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} \underline{\underline{y = 3\cos(\sqrt{3}\,t) - \cos t}}$$

b) $y(t) + e^{-3t} * y(t) = 2t^2$

$$\overset{\mathcal{L}}{\Longrightarrow} Y + \frac{1}{s+3} \cdot Y = 2 \cdot \frac{2}{s^3} = \frac{4}{s^3}$$

$$\Longleftrightarrow Y = \frac{4}{s^3} \cdot \frac{1}{1 + 1/_{s+3}}$$

$$= \frac{4}{s^3} \cdot \frac{s+3}{s+3+1}$$

$$= \frac{4}{s^3} \cdot \frac{s+3}{s+4}$$

$$= \frac{4s+12}{s^4 + 4s^3}$$

$$= \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s+4}$$

$$\Longrightarrow A = 3,\ B = 0,\ C = 0,\ D = 1/_{16}$$

$$\Longrightarrow Y = 3 \cdot \frac{1}{s^3} + \frac{1}{16} \cdot \frac{1}{s+4}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} \underline{\underline{y = \frac{3}{2}t^2 + \frac{1}{16}e^{-4t}}}$$

c) $L = 0.5,\ R = 2,\ C = 1,\ i_0 = 0$ in

$$L i'(t) + R i(t) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = \delta(t)$$

$$\Leftrightarrow L i'(t) + R i(t) + \frac{1}{C}(i * 1) = \delta(t)$$

$$\xrightarrow{\mathcal{L}} L(sI - i_0) + RI + \frac{1}{C} I \cdot \frac{1}{s} = e^{-s\cdot 0}$$

$$\Leftrightarrow LsI - Li_0 + RI + \frac{1}{Cs} I = 1$$

$$\Leftrightarrow I = (1 + Li_0) : \left(Ls + R + \frac{1}{Cs}\right)$$

$$= \frac{Cs(1 + Li_0)}{CLs^2 + CRs + 1} \longrightarrow 1 \cdot s \cdot \left(1 + \tfrac{1}{2}\cdot 0\right) = s$$

$$CLs^2 + CRs + 1 \to \frac{1}{2}s^2 + 2s + 1 = 0$$

$$\Rightarrow s^2 + 4s + 2 = 0$$

$$\Rightarrow (s+2)^2 - 2 = 0$$

$$\Rightarrow (s + 2 + \sqrt{2})(s + 2 - \sqrt{2}) = 0$$

$$\Rightarrow I = \frac{2s}{(s + 2 + \sqrt{2})(s + 2 - \sqrt{2})}$$

Could use abc but I'm intimidated by the algebra

From the transformation table we have

$$ae^{at} - be^{bt} \xrightarrow{\mathcal{L}} \frac{(a-b)s}{(s-a)(s-b)}.$$

Let $a = -2 - \sqrt{2}$ and $b = -2 + \sqrt{2}$, then

$$I = -\frac{1}{\sqrt{2}} \cdot \frac{-2\sqrt{2}\, s}{(s+2+\sqrt{2})(s+2-\sqrt{2})}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} i = -\frac{\sqrt{2}}{2}\left([-2-\sqrt{2}] e^{[-2-\sqrt{2}]t} - [-2+\sqrt{2}] e^{[-2+\sqrt{2}]t} \right)$$

$$= \frac{\sqrt{2}}{2}\left[(2+\sqrt{2}) e^{(-2-\sqrt{2})t} + (-2+\sqrt{2}) e^{(-2+\sqrt{2})t} \right]$$

$$= (\sqrt{2}+1) e^{(-2-\sqrt{2})t} + (-\sqrt{2}+1) e^{(-2+\sqrt{2})t}$$

Problem 3

a) $y'' + 3y' + 2y = \delta(t)$

$$\overset{\mathcal{L}}{\Longrightarrow} s^2 Y - s y_0 - y_0' + 3(sY - y_0) + 2Y = 1$$

$$\Leftrightarrow s^2 Y - s y_0 - y_0' + 3sY - 3y_0 + 2Y = 1$$

$$\Leftrightarrow Y = (1 + s y_0 + y_0' + 3y_0) \div (s^2 + 3s + 2),$$

$$s^2 + 3s + 2 = (s+2)(s+1) = 0$$

$$\Rightarrow Y = \frac{s y_0 + y_0' + 3y_0 + 1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$\Longrightarrow A = 2y_0 - y_0' - 3y_0 - 1 = -(y_0 + y_0' + 1),$$

$$B = -y_0 + y_0' + 3y_0 + 1 = y_0' + 2y_0 + 1$$

$$\Rightarrow Y = \frac{y_0' + 2y_0 + 1}{s+1} - \frac{y_0 + y_0' + 1}{s+2}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} y = (y_0' + 2y_0 + 1) e^{-t} - (y_0 + y_0' + 1) e^{-2t}$$

Let $y_0 = 0$ and $y_0' = 1$,

$$\left.\begin{aligned} & y = 2e^{-t} - 2e^{-2t} \\ \Rightarrow\ & y' = -2e^{-t} + 4e^{-2t} \\ \Rightarrow\ & y'' = 2e^{-t} - 8e^{-2t} \end{aligned}\right\} \quad \begin{aligned} & y'' + 3y' + 2y \\ & = (2 + 3\cdot(-2) + 2\cdot 2)\, e^{-t} \\ & + (-8 + 3\cdot 4 + 2\cdot(-2))\, e^{-2t} = 0 \end{aligned}$$

Since our solution for $y'' + 3y' + 2y = \delta$ worked for $y'' + 3y' + 2y = 0$ given the initial values, we see that this y-solution is common to both ODEs.

b) Let $y_0 = y'_0 = 0$ and $y''_0 = 1$ in

$$y''' + 3y'' + 3y' + y = 0$$

$$\overset{\mathcal{L}}{\Longrightarrow} s^3 Y - s^2 \cancel{y_0} - s \cancel{y'_0} - y''_0$$
$$+ 3(s^2 Y - s\cancel{y_0} - \cancel{y'_0})$$
$$+ 3(sY - \cancel{y_0}) + Y = 0$$

initial

$$\Leftrightarrow s^3 Y - 1 + 3s^2 Y + 3sY + Y = 0$$

$$\Leftrightarrow Y(s^3 + 3s^2 + 3s + 1) = 1$$

$$\Leftrightarrow Y(s+1)^3 = 1$$

$$\Leftrightarrow Y = \frac{1}{(s+1)^3}$$

$$\overset{\mathcal{L}^{-1}}{\Longrightarrow} y = e^{-t} * e^{-t} * e^{-t}$$

$$= e^{-t} * \int_0^t e^{-\tau} \cdot e^{-(t-\tau)} d\tau$$

$$= e^{-t} * \int_0^t e^{\cancel{-\tau} - t + \cancel{\tau}} d\tau$$

$$= e^{-t} * \int_0^t \overbrace{e^{-t}}^{\text{const.}} d\tau$$

$$= e^{-t} * (te^{-t}) = (te^{-t}) * e^{-t}$$

$$= \int_0^t \tau e^{-\tau} e^{-(t-\tau)} d\tau$$

$$= \int_1^t \tau e^{-t}\, d\tau$$

$$= \frac{1}{2} t^2 e^{-t}$$

Check it:

$$y' = te^{-t} - \frac{1}{2} t^2 e^{-t} = te^{-t} - y,$$

$$y'' = e^{-t} - te^{-t} - y',$$

$$y''' = \cancel{-e^{-t}} - \cancel{(e^{-t}} - te^{-t}) - y''$$

$$= -te^{-t} - y''$$

$$\longrightarrow (-te^{-t} - y'') + 3(e^{-t} - te^{-t} - y')$$

$$+ 3(te^{-t} - y) + \frac{1}{2} t^2 e^{-t}$$

$$= (\cancel{-te^{-t}} - [e^{-t} \cancel{- te^{-t}} - y'])$$

$$+ 3(e^{-t} - te^{-t} - [te^{-t} - y])$$

$$+ 3(te^{-t} - \frac{1}{2} t^2 e^{-t}) + \frac{1}{2} t^2 e^{-t}$$

$$= (-e^{-t} - [te^{-t} - y])$$

$$+ 3(e^{-t} - 2te^{-t} - [\frac{1}{2} t^2 e^{-t}])$$

$$+ 3te^{-t} - \frac{3}{2} t^2 e^{-t} + \frac{1}{2} t^2 e^{-t}$$

$$= -e^{-t} - te^{-t} - \frac{1}{2} t^2 e^{-t}$$

$$+3e^{-t} - 6te^{-t} - \frac{3}{2}t^2e^{-t}$$

$$+3te^{-t} - \frac{3}{2}t^2e^{-t} + \frac{1}{2}t^2e^{-t}$$

$\neq 0$, so y is not a solution and I have made an error.

Problem 4

Let $g(t) = \begin{cases} 0, & t < 2 \\ t, & 2 \leq t \leq 5 \\ 0, & 5 < t \end{cases}$

$$\Rightarrow g(t) = u(t-2)\,t - u(t-5)\,t$$

$$= u(t-2)\cdot((t-2)+2) - u(t-5)\cdot((t-5)+5)$$

$$= u(t-2)\cdot(t-2) + 2u(t-2)$$

$$- u(t-5)\cdot(t-5) - 5u(t-5)$$

$$\Rightarrow \mathcal{L}\{g(t)\}(s) = \left(\frac{1}{s^2} + \frac{2}{s}\right)\cdot e^{-2s}$$
$$- \left(\frac{1}{s^2} + \frac{5}{s}\right) e^{-5s}$$

$$\mathcal{L}\{f''(t) - f(t)\}(s)$$

$$= s^2 F(s) - s\cancel{f(0)} - \cancel{f'(0)} - F(s)$$

$$= F(s)\cdot[s^2 - 1]$$

$$\Rightarrow \mathcal{L}\{f''(t) - f(t)\} = \mathcal{L}\{g(t)\}$$

$$\Leftrightarrow F(s)\,[s^2 - 1] = \left(\frac{1}{s^2} + \frac{2}{s}\right)e^{-2s} - \left(\frac{1}{s^2} + \frac{5}{s}\right)e^{-5s}$$

$$\Leftrightarrow F(s) = \frac{(1 + 2s)e^{-2s} - (1 + 5s)e^{-5s}}{s^4 - s^2}$$

$$\frac{1+2s}{s^4-s^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1}$$

$$\Leftrightarrow \quad 1+2s = A(s^3-s) + B(s^2-1) + Cs^3 + Ds^2$$

But we also have $\frac{1+5s}{s^4-s^2}$, so

$$\Rightarrow \quad 1+5s = A(s^3-s) + B(s^2-1) + Cs^3 + Ds^2$$

$$\Rightarrow \begin{array}{c} 1 \\ s \\ s^2 \\ s^3 \end{array} \left[\begin{array}{cccc|cc} 0 & -1 & 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 0 & 2 & 5 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & -2 & -5 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow$$

- $$\frac{1+2s}{s^4-s^2} = -\frac{2}{s} - \frac{1}{s^2} + \frac{2s+1}{s^2-1}$$

- $$\frac{1+5s}{s^4-s^2} = -\frac{5}{s} - \frac{1}{s^2} + \frac{5s+1}{s^2-1}$$

Let D denote 5 or 2 in the above

$$\overset{\mathcal{L}}{\Rightarrow} -D - t + D\cosh t + \sinh t \text{, thus}$$

$$\Rightarrow f(t) = \left(-2-(t-2) + 2\cosh(t-2) + \sinh(t-2)\right) - \left(-5-(t-5) + 5\cosh(t-5) + \sinh(t-5)\right)$$

$$= 12 + 2\cosh(t-2) + \sinh(t-2) + 5\cosh(t-5) + \sinh(t-5)$$